SHENTON COLLEGE

Mathematics Methods Year 11

2016 Test 4

NAME: Solutions

Teacher (circle one): Friday Mackenzie McRae

Section 1: Calculator Free (No notes, formula sheet) **(25 minutes, marks)**

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QUESTION 1 [1, 2, 2, 3 = 8 marks]

Evaluate where possible or otherwise simplify (resulting in **positive** indices) the following:

(a) $25^{\frac{3}{2}}$

$= (5^2)^{\frac{3}{2}}$

$= 125$ ✓

(b) $\frac{(p^2)^0}{(3p)^2}$ ✓

$= \frac{1}{9p^2}$ ✓

* correct use of index law -1 not +ve.

(c) $\left(\frac{x^4y}{xy^3}\right)^{-2} = \frac{x^{-8}y^{-2}}{x y^{3}}$ ✓

$= \frac{y^4}{x^6}$ ✓

(d) $\frac{(a^3b^{-2})^4}{\sqrt{a^2b^4}} = \frac{a^{12}b^{-8}}{(a^2b^4)^{\frac{1}{2}}}$ ✓

$= \frac{a^{12}b^{-8}}{a b^2}$ ✓

$= \frac{a^{11}}{b^{10}}$ ✓

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QUESTION 2 [2, 2, 3, 2 = 9 marks]

Solve the following showing all working:

a) $2a^3 - 1 = 127$

$2a^3 = 128$ ✓ process

$a^3 = 64$

$a = 4$ ✓ answer

b) $3^{n-2} = 81$

$3^{n-2} = 3^4$ ✓ process

$\therefore n - 2 = 4$

$n = 6$ ✓ answer

c) $2^{2x} - 3 \times 2^x + 2 = 0$

$(2^x - 2)(2^x - 1) = 0$ ✓ process

$\therefore 2^x = 2, \; x = 1$ ✓ answer

$\therefore 2^x = 1, \; x = 0$ ✓ answer

d) $4^{3x+1} = \frac{1}{8}$

$2^{6x+2} = 2^{-3}$ ✓ process

$\therefore 6x + 2 = -3$

$x = -\frac{5}{6}$ ✓ answer

Question 3. [3 marks] Badly done!

If the angles of a triangle are in arithmetic progressions, use working to show that one of the angles must be $60°$ in size.

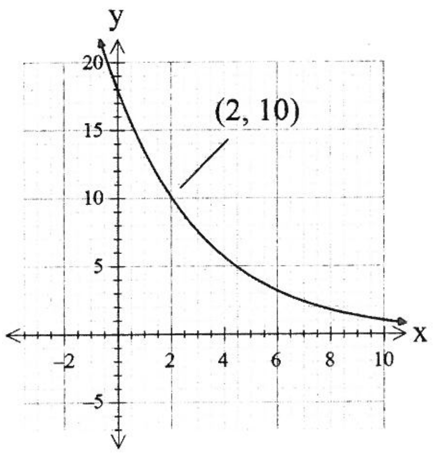
$$a + (a+d) + (a+2d) = 180$$ ✓ equation correct

$$3a + 3d = 180$$ ✓ simplifying

$$\therefore a+d = 60$$ ✓ recognition of angle

(middle size angle)

Question 4. [3, 2, 1 = 6 marks]



The exponential graph on the left has a y intercept of 18 and passes through the point (2, 10).

a) Find the equation of this function, leaving your answer with exact values.

$$y = ab^x$$

$(0,18) \therefore a = 18$ ✓

$(2,10)$ $18b^2 = 10$

$$b^2 = \frac{10}{18}$$

$$b = \frac{\sqrt{5}}{3}$$ ✓ process

$$\therefore y = 18\left(\frac{\sqrt{5}}{3}\right)^x$$ ✓

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b) What is the domain and range of this function?

$D: \{x : x \in R\}$ ✓

$R: \{y : y > 0\}$ ✓

c) If the function is translated down 5 units and then reflected about the x axis, what would be the new y intercept?

−13

$$(0,18) \rightarrow (0,13) \rightarrow (0,-13)$$ ✓ answer

Question 5. [3 marks]
Show using first principles how to determine the gradient function of $y = 2x^2 - 3x$

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{2(x+h)^2 - 3(x+h) - 2x^2 + 3x}{h}$$ ✓ substitution and rule

$$= \lim_{h \to 0} \frac{\cancel{2x^2} + 4xh + 2h^2 - 3x - 3h - \cancel{2x^2} + \cancel{3x}}{h}$$ ✓ cancelling and process

$$= \lim_{h \to 0} \frac{\cancel{h}(4x + 2h - 3)}{\cancel{h}} = 4x - 3$$ ✓ answer

Some separated the functions? 3 marks?

Question 6. [4 marks]
Sketch the graph of a function that satisfies all the conditions stated below

- The functions meets the x axis at (-2,0) ✓
- The function has a positive gradient when $x > -2$ and negative gradient for $x < -2$ ✓
- The gradient of the function is zero when $x = -2$ and $x = 3$ ✓
- The y intercept is positive ✓

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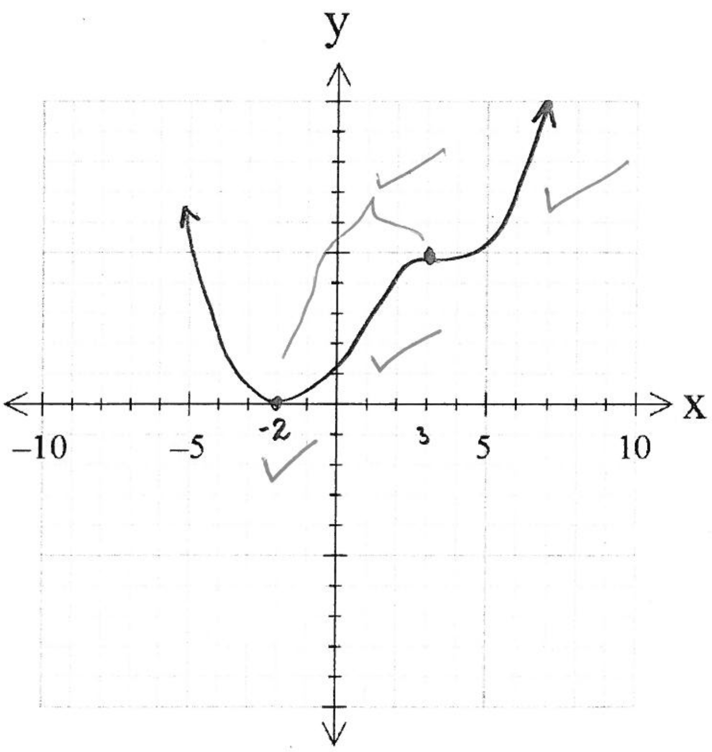

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Section 2: Calculator (1 page of notes, 1 side; formula sheet) **(20 minutes, 28 marks)**

Question 1. [2, 2, 3 & 2 = 9 marks]

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Two sequences A and T are defined below.

$T_n = 100 - 2n$

$A_n = 0.8A_{n-1}$ $A_3 = 4$

(a) Find the first 4 terms of both sequences.

T 98, 96, 94, 92 ✓

A 6.25, 5, 4, 3.2 ✓

(b) Write a recursive definition for T_n.

$T_{n+1} = T_n - 2$, $T_1 = 98$ ✓ ✓ $T_0 = 100$

(c) The sum of ~~One~~ one of sequences tends towards a certain value. What is this value and explain why it does this?

A sequence $S_\infty = \frac{6.25}{1-\frac{4}{5}} = 31.25$ ✓ ✓

It is a geometric decay sequence. ✓

(d) Calculate the sum of the terms T_{40} to T_{60}, inclusive.

$S_{60} - S_{39}$ ✓

$\therefore 2340 - 2340$

$= 0$ ✓

Question 2. [2, 2, 3, 2, 2 = 11 marks]

The population of Llamas in a South American reserve is slowly dwindling due to new management. After 3 years the population of Llamas is 1244 and two years later the population is 876. If the population is declining at an exponential rate

a) What percentage of Llamas are they losing per year (to 1 d.p.)?

$1244 \times r^2 = 876$ ✓

$\therefore r = 0.8392$

$\therefore 16.1\%$ ✓

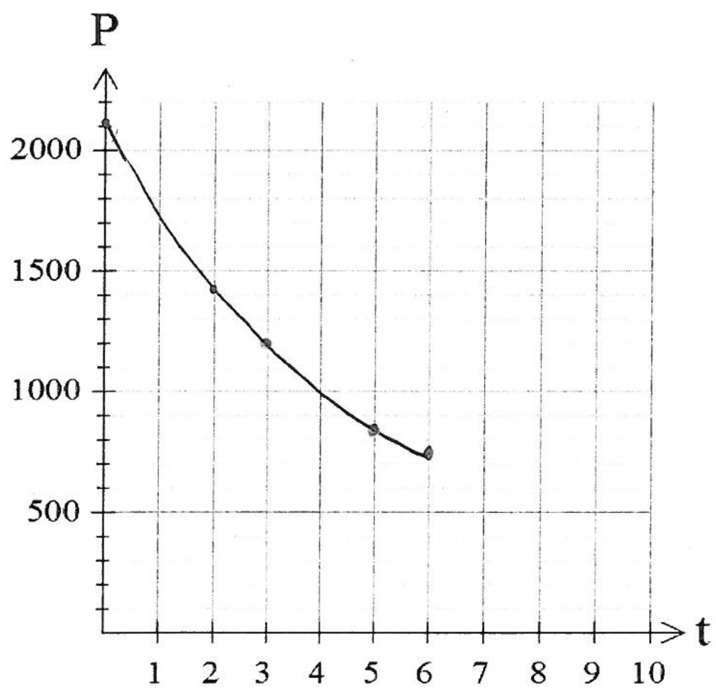
b) How many Llamas were there when the new management took over?

$\frac{1244}{r^3}$ ✓

$a = 2105$ ✓

c) Use the grid below to draw a graph of the population of Llamas after new management took over, for $0 \le t \le 6$, where t is the time in years.

use follow through.



✓ domain

✓ y intercept

✓ shape and accuracy

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d) Write a general rule in terms of years (t) describing the population (P) of the Llamas after new management began.

$T_n = 2105\,(0.8392)^t$ ✓ ✓

e) After 6 years the current management is fired and a breeding program is developed that promises that numbers will be back up to the original level in 4 years' time. What percentage growth rate must they have promised?

$735.14 \times x^4 = 2105$ ✓ process

$x = 1.3008$

$\therefore 30.1\%$ ✓ answer

2001	2000 2000	$\times 1.038$	2076
2002	2076, 3076	$\times 1.038$	3192.88
2003	3192.88, 4192.88	$\times 1.038$	4352.22
2004	4352.22, 5352.22	$\times 1.038$	

Question 3. [2, 2 = 4 marks]

On the 1st January 2001 John opens an account for his new born baby boy with a deposit of \$2000 in an account that accrues interest at 3.8% compounded annually. On the same day each year he puts in another \$1000 into the account. If the interest rate stays the same for the time he has the account

a) Write a recursive rule that describes this investment.

$T_{n+1} = 1.038T_n + 1000$, ✓ rule

$T_0 = 2000$ ✓ starting point

b) How much will he have in the account if he closes the account after 12 years, just before he makes his annual January deposit?

$T_{12} = \$17983.52$ ✓

$\therefore$ \$16 983.52 before deposit ✓

or \$16361.77 then $\times 1.038 =$

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Question 4. [4 marks]

G.P. series $40 + 24 + 14.4 \ldots$ least value of n so that S_∞ and S_n difference < 0.2.

$a = 40$

$r = 0.6$ ✓

$S_\infty = \dfrac{40}{1-0.6} = 100$ ✓

SOLVE

$n = 12.17$ ✓

In sequence,

n	Series
12	99.782
13	99.869
14	99.922

✓

$\therefore n = 13$ ✓